**Problem 1:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| World | X | Y | Z | (𝑋∨𝑌∨𝑍) | (¬𝑌∨¬𝑍) | (𝑋∨𝑌∨𝑍)∧(¬𝑌∨¬𝑍) |
| w1 | T | T | T | T | F | F |
| w2 | T | T | F | T | T | T |
| w3 | T | F | T | T | T | T |
| w4 | T | F | F | T | T | T |
| w5 | F | T | T | T | F | F |
| w6 | F | T | F | T | T | F |
| w7 | F | F | T | T | T | T |
| w8 | F | F | F | F | T | F |

1. Let G = (X∨Y∨Z)∧(¬Y∨¬Z)⊭(X). If (X v Y v Z ) && (¬Y v Z) ⊭ X, then that means that M((X v Y v Z ) && (¬Y v Z)) is not a subset of X, which is true as seen in the model above. M(X) = {w1,w2, w3, w4}, while M(G) = {w2,w3,w4,w7}. M(G) is not a subset of M(X) because M(G) contains w7, while M(X) does not.
2. Having Horn clauses would be useful because it would be easy to cancel out a single variable since there is at most 1 positive proposition. We can easily use resolution without getting vacuous clauses.

**Problem 2:**

1. Create a KB by converting each of the above English sentences into a propositional logic one.
   1. Each propositional logic sentence is enumerated, and in the KB they are ∧ together.
   2. KB = {
      1. Forneybots were found to malfunction [M] if and only if they suffer water damage [D] or overheard a logical paradox [P].
         1. M ⇔ D v P
      2. Forneybots may have been caught in a sudden rainstorm [R], which are known to happen only during Winter [W] or Summer [S], though not all Winters and Summers had rain. R ⇒ W v S
         1. R ⇒ W v S
      3. It also goes without saying that if it is one season, it cannot simultaneously be the others (in this dystopian future, Spring is no longer a season, so don't worry about it).
         1. W⇔ ¬ S ∧ ¬F
         2. S ⇔ ¬W ∧ ¬F
         3. F ⇔ ¬ S ∧ ¬W
      4. Moreover, rainstorms and water damage would have gone hand in hand, since the Forneybots were not aware of their weakness.
         1. R ⇔ D
      5. Logical paradoxes, on the other hand, would have only been heard during election season (through TV or radio broadcast), which are held in the Fall [F] (and thus, all Fall seasons came hand in hand with paradoxes).
         1. P ⇔ F
      6. The Great Forneybot Uprising occurred from a mass malfunction on Nov. 16, 2024 (note the season).
         1. M< ⇒ F
   3. In other words

KB = {

(M ⇔ D v P) ∧( R ⇒ W v S) ∧(W⇔ ¬ S ∧ ¬F) ∧ (S ⇔ ¬W ∧ ¬F) ∧ (F ⇔ ¬ S ∧ ¬W) ∧ (R ⇔ D) ∧ (P ⇔ F) ∧ (M ⇔ F)

}

1. Convert the KB into CNF, and enumerate each clause.
   1. (M ⇔ D v P) ≡ (M ⇒ D v P) ∧ (D v P ⇒ M) ≡ (¬M v D v P) ∧ (¬(D v P) v M) ≡ (¬M v D v P) ∧ ((¬D ∧ ¬P) v M ≡ **(¬M v D v P) ∧ (¬D v M) ∧ (¬P v M)**
   2. ( R ⇒ W v S) ≡ **( ¬R v W vS)**
   3. (W⇔ ¬ S ∧ ¬F) ≡ (W ⇒ ¬S ∧ ¬F) ∧ (¬S ∧¬F ⇒ W) ≡ (¬W v (¬S ∧F)) ∧ (¬(¬S∧¬F) v W) ≡ **(¬W v ¬S) ∧(¬W v¬F)∧(S v F v W)**
   4. (S ⇔ ¬W ∧ ¬F) ≡ **(¬S v ¬W) ∧(¬S v¬F)∧(W v F v S)** 
      1. See C and replace variables accordingly
   5. (F ⇔ ¬ S ∧ ¬W) ≡ **(¬F v ¬S) ∧(¬F v¬W)∧(S v W v F)** 
      1. See c and replace variables accordingly
   6. (R ⇔ D) ≡ (R ⇒ D) ∧ (D ⇒ R) ≡ **(¬R v D) ∧ (¬D v R)**
   7. (P ⇔ F) ≡ (P ⇒ F) ∧ (F ⇒ P) ≡ **(¬P v F) ∧ (¬F v P)**
   8. (M ⇔ F) ≡ (M ⇒ F) ∧ (F ⇒ M) ≡ **(¬M v F) ∧ (¬F v M)**
   9. In other words

KB = {

(¬M v D v P) ∧ (¬D v M) ∧ (¬P v M) ∧( ¬R v W vS)∧(¬W v ¬S) ∧(¬W v¬F)∧(S v F v W) ∧(¬S v ¬W) ∧(¬S v¬F)∧(W v F v S)∧ (¬F v ¬S) ∧(¬F v¬W)∧(S v W v F) ∧ (¬R v D) ∧ (¬D v R)∧ (¬P v F) ∧ (¬F v P)∧(¬M v F) ∧ (¬F v M)

}

1. Using the above, we must settle the debate by showing that one of the following queries is true: (1) α=¬D∧P (i.e., that it was an overheard paradox and NOT water damage that caused the uprising) or (2) β=¬P∧D (vice versa).
   1. KB = {
      1. (¬M v D v P) ∧
      2. (¬D v M) ∧
      3. (¬P v M) ∧
      4. ( ¬R v W vS) ∧
      5. (¬W v ¬S) ∧
      6. (¬W v¬F) ∧
      7. (S v F v W) ∧
      8. (¬S v ¬W) ∧
      9. (¬S v¬F) ∧
      10. (W v F v S) ∧
      11. (¬F v ¬S) ∧
      12. (¬F v¬W) ∧
      13. (S v W v F) ∧
      14. (¬R v D) ∧
      15. (¬D v R) ∧
      16. (¬P v F) ∧
      17. (¬F v P) ∧
      18. (¬M v F) ∧
      19. (¬F v M) ∧
      20. ————————————- Temp( Query α=¬D∧P) Suppose ¬α = Dv¬P
      21. D v ¬P ∧
      22. R v ¬P (3.1.21 and 3.1.15) ∧
      23. R v ¬F (3.1.22 and 3.1.17) ∧
      24. R v ¬M(3.1.23 and 3.1.18) ∧
      25. D v ¬M(3.1.24 and 3.1.14) ∧
      26. Ø (3.1.25 and 3.1.2)
      27. ————————————— Temp Ended in Ø therefore, α = true
      28. ¬D ∧
      29. P }
   2. In other words, it was an overheard paradox and NOT water damage that caused the uprising